Discrete Random Variables

4.4 Binomial Distribution and Binomial Probability

1. Each year in the United States, S&E (science and engineering) degrees are one-third of the awarded bachelor's degrees.
   a. What is the probability that among 16 recent graduates half of them have a S&E degree?
   b. What is the probability that at most half of them have S&E degrees?

2. Selective schools generally have higher graduation rates than nonselective schools for students of all races attending university in the United States. The graduation rate for selective schools is approximately 80% (compared to just 59% at nonselective schools).
   a. Using this information, what is the probability that among ten randomly selected students attending a selective university in the US, exactly seven of them graduate?
   b. What is the probability that at least seven of the ten graduate?

3. Use the information contained within the following excerpt of a NY Times article published in 2006 to answer the questions below: "About half of Americans born at the turn of the 20th century had blue eyes, according to a 2002 Loyola University study in Chicago. By mid-century that number had dropped to a third. Today only about one 1 of every 6 Americans has blue eyes, said Mark Grant, the epidemiologist who conducted the study."
   a. What is the probability that among five randomly selected people today two of them have blue eyes?
   b. What is the probability that more than two of the five have blue eyes?

Answers:

1. a. P(X = 8) = $_{16}C_8\left(\frac{1}{3}\right)^8\left(\frac{2}{3}\right)^8$ = 0.0765

   b. $P(X \le 8) = {}_{16}C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^{16} + {}_{16}C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^{15} + \ldots + {}_{16}C_8\left(\frac{1}{3}\right)^8\left(\frac{2}{3}\right)^8$ = 0.950

2. a. $P(X = 7) = {}_{10}C_7\,(0.8)^7\,(0.2)^3$ = 0.201

   b. $P(X \ge 7) = {}_{10}C_7\,(0.8)^7\,(0.2)^3 + {}_{10}C_8\,(0.8)^8\,(0.2)^2 + \ldots + {}_{10}C_{10}\,(0.8)^{10}\,(0.2)^0$ = 0.879

3. a. $P(X = 2) = {}_5C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 = 0.161$

   b. About 3.6%

   $P(X > 2) = {}_5C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2 + {}_5C_4\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^1 + {}_5C_5\left(\frac{1}{6}\right)^5\left(\frac{5}{6}\right)^0 = 0.0355$